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Mathematics in the news and prospective teachers' professional mathematical literacy

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In this article, the authors describe how they helped their prospective teachers to understand numeracy by writing open-ended mathematical tasks based on articles from the news. 'Mathematics in The News' may be a useful rich context for teachers to consider using in their school mathematics classes, with the potential to prompt students' mathematical thinking.

Introduction

Mathematical literacy, or numeracy, is generally accepted in Australia as one of the goals of education (Australian Curriculum, Assessment, and Reporting Authority (ACARA), n.d.) and internationally (Organisation for Economic Co-operation and Development (OECD), 1999). Mathematical literacy is the capacity to:

> "... identify, to understand and to engage in mathematics and make well-founded judgements about the role that mathematics plays, as needed for an individual's current and future life, occupational life, social life with peers and relatives, and life as a constructive, concerned and reflective citizen." (OECD, 1999, p. 50).

Teachers can support their students in developing their mathematical literacy but to do this, teachers need professional mathematical literacy (Oonk, Van Zanten, & Keijzer, 2007). Professional mathematical literacy includes deep knowledge of meaningful mathematics and how to use this knowledge when teaching mathematics to students (Ball, Thames, & Phelps, 2008).

In this article, we discuss a project with our prospective teachers where we helped them to develop their understanding of numeracy by writing open-ended mathematical tasks based on articles from the news. We suggest that this may be useful for classroom teachers to consider for their students.

Prospective primary school teachers' professional mathematical literacy is operationalised in the Dutch mathematics knowledge base (Van Zanten, Barth, Faarts, Van Gool, & Keijzer, 2009). Moreover, professional mathematical literacy is tested in the so-called knowledge base test (Landelijke kennistoets Wiskunde), which all prospective teachers in primary teacher education must pass while still at university before they can be registered as teachers. Teacher education curricula therefore includes mathematical content knowledge to enable prospective teachers to attain mastery of mathematics at a level well above the level of mathematics they are teaching in primary school. However, in doing so the curricula tend to isolate content knowledge from pedagogical content knowledge (Keijzer, 2019).

In this paper, we describe how we tried to relate content knowledge and aspects of pedagogical content knowledge in the iPabo University of Applied Sciences (a primary school teacher education institute in Amsterdam, the Netherlands) mathematics curriculum. We, the authors of this paper, are teacher educators and researchers. As teacher educators, we prepared prospective teachers' professional mathematical literacy not by providing isolated mathematics problems, but by asking them to look for mathematics in the news. Their main assessment task consisted of developing an open-ended mathematics problem based on any newspaper article published during assessment day. As researchers we then assessed the effectiveness of this style of intervention and assessment strategy.

Context

Sixty-five first year prospective teachers (median age 19 [17–21]; 54 women and 11 men) from three first year groups participated in this study. Over the course of three one-hour meetings, in the first lessons, at the very start of the school year, these prospective teachers focused on the following steps, aimed at developing both their mathematical content knowledge and aspects of their pedagogical content knowledge:

1. choose a meaningful news item suitable for primary school students,
2. formulate at least one related open ended mathematics problem,
3. give clear arguments for the chosen news item being suitable for the target group,
4. reflect on a range of possible mathematical strategies for solving the problem.

In this paper, we focus on the second step: formulating open-ended mathematics problems. As this took place in the very first meetings of their teacher education, the prospective teachers had no previous experience on creating real-world mathematical tasks for primary-aged children.

University meetings

In the first meeting, prospective teachers and the teacher educator selected news items from newspaper websites and discussed these, followed by the educator sharing examples on formulating open-ended mathematics problems. This was probably the first time prospective teachers looked at news items from a mathematical point of view. In preparation for the second meeting, the prospective teachers were instructed to individually search for a news item and construct a related open-ended mathematics problem. In doing so prospective teachers came up with mathematics problems like 'What would be the average amount of money spent by Dutch inhabitants on French fries in his or her life?' This problem requires reflecting on a number in the news, which means mathematising the situation, using personal references, and making calculations, facilitating, and reflecting on the situation.

During the second meeting, the mathematics problems prospective teachers prepared were discussed in class. Many prospective teachers formulated rather closed problems. After a round of feedback in which the teacher educator focused on improving students' proposed open ended problems, prospective teachers successfully formulated open problems, which now also related to themselves. Typical problems designed in this round included a problem like 'How many kilos of meat would you save if you did not eat meat for a whole year?'

The preparation for the third meeting was to go through all the steps in a group of four or five prospective teachers, namely, select a news item, come up with an open-ended problem, select the target group, solve the problem in two different manners, and reflect on this process. In the third and final meeting, prospective teachers shared their group work and received feedback from two other groups and their teacher educator.

These meetings were regular classes for all prospective teachers.

Assessment task

Activities at the university were followed by a similar assessment task (Figure 1). Prospective teachers worked in groups on the assessment task. They were allowed 70 minutes to complete the task at the institute, however working on the task could have started in the 24 hours before.

> Find a news item published in the last 24 hours. Design an open ended mathematical problem, related to the article, suitable for students aged 4 to 12.

Figure 1. Design task as an assessment task.

Analysis

The quality of the open-ended mathematics problems designed by the groups was analysed. Doing so we focused on three aspects of the mathematics problems: problem openness, the mathematical complexity of the problem, and the problem's artificiality.

The researchers coded the prospective teachers' designs in two rounds on three-point scales. Individual coding was then followed by comparing codes. When codes differed between researchers, they were discussed together until unanimity was reached. Each problem's openness was coded: closed (1), somewhat open (2), and, open (3). A problem's mathematical complexity was coded according to the type of mathematical thinking required for solving the problem: reproduction, definitions, computations (1), connections and integration for problem solving (2),

Results

Ten out of thirteen groups of prospective teachers participated in a follow-up session. Herein, we asked prospective teachers to categorise their designed problems and reflect on what they learned. Table 1 shows both educators' and prospective teachers' categorisation.

Aspects		Number of problems	
		Categorised by educators	Categorised by prospective teachers
Openness	1. Closed 2. Somewhat open 3. Open	4 5 1	0 1 9
Mathematical complexity	1. Reproduction 2. Connection 3. Mathematisation	1 4 5	1 5 4
Artificiality	1. Artificial 2. Somewhat artificial 3. Not artificial	5 1 4	

Table 1. Number of problems categorised by teacher educators and prospective teachers on three aspects related to the intrinsic quality of the mathematics problems prospective teachers designed.

Teacher educators' perspective

When considering the problem's openness, the teacher educators identified that only one prospective teacher group formulated an open-ended problem, which elicited mathematical thinking. Example 1 shows this problem:

Example 1

Seventy trucks (see Figure 2) emptied their loads of sand on a square. How many sandboxes could be filled with this amount of sand? [Educators coded: open, mathematisation, not artificial]

Figure 2. One of the trucks unloading the sand (De Wit, 2019).

This problem was categorised as 'open, mathematisation, not artificial' because solving the problem requires prospective teachers making assumptions about the context. For example, solving this problem involves estimating the amount of sand that fits in a regular truck and the size of a sandbox, and choosing one's own problem-solving approach. According to the educators most problems prospective teachers formulated were less open (four were closed and five were a bit open).

Example 2 typifies a less open style of problem, and although the size of the bar is unspecified, all the necessary information is provided and only one solution method is possible.

Example 2

Trick or treating. You bring a bag with a length of 30 cm, width of 20 cm, and height of 10 cm. How many chocolate bars fit in the bag? [Educators coded: closed, reproduction, artificial]

In terms of complexity, half the prospective teacher groups designed problems in which mathematisation was clearly elicited (five out of ten groups). Example 1 provides such a problem. The problem formulated by one group required only straightforward reproduction or calculations (Example 2). Four groups designed problems in which some connections or assumptions about the context were needed to solve the problem:

Example 3

If you align all Play Stations sold; how many kilometres of Play Stations will this result in? [Educators coded: somewhat open, connection, artificial]

This question was classified as artificial. Prospective teachers found it difficult to generate non-artificial questions. They often used the same question-type for different contexts regardless of its appropriateness, for example, stacking a pile of a certain object. About half of the questions were classified as either artificial (five) or not artificial (four), only one question was in between.

Prospective teachers' perspective

The categorisation by the prospective teachers differed somewhat from that by the teacher educators (see Table 1). Prospective teachers classified the openness of their problems of equal quality (two) or higher quality (eight). Mathematical complexity was classified almost always equal (nine) by the prospective teachers and the teacher educators.

In the follow-up meeting we also discussed whether the prospective teachers considered the assignment as useful to include in next year's curriculum and whether and how it should be changed. Most of the

prospective teachers appreciated the assignment: "it gives me a lot of inspiration for my own classroom", "it's a challenging assignment"or "it may be difficult to reflect on different solutions, but this assignment makes it easier."

When asked what they learned from the assignment, prospective teachers described that they learned "to convert a news item into a calculation assignment", "to formulate an open-ended problem", or "to convert a current topic into a meaningful mathematics assignment for primary school students."

Final reflections

The prospective teacher assignment described in this paper was challenging for first-year prospective teachers, in terms of both their mathematical knowledge and pedagogical content knowledge. After participating in the Mathematics in The News meetings most prospective teachers were able to formulate open-ended mathematical problems. In these problems, on the one hand they challenged their (hypothetical) primary school students to choose their own mathematical approach and on the other hand to explore different mathematical reasoning skills.

During the group discussion in the evaluation of the assignment, prospective teachers indicated that they did not think their open-ended mathematics problems to be directly useful for their own primary school students. They described that these open-ended mathematical problems would, in their opinion, be too difficult, because such problems are not included in the regular primary school mathematics textbooks (see e.g., Van Zanten & Van den Heuvel-Panhuizen, 2018). Important to note is that the prospective teachers had only had a few days of experience in their primary school class and using news items is not customary in primary school mathematics education (Fase, 2011). This made it difficult for them to estimate what to expect from the primary school students. Including this assignment at the very start of their teacher education is challenging. In the remainder of their studies, prospective teachers will gain much more experience with creating authentic mathematical tasks, but these first experiences might be highly formative.

The primary goal of the Mathematics in The News assignment was for prospective teachers to relate their mathematical content knowledge and aspects from pedagogical content knowledge. The problems prospective teachers designed showed how they integrated these two types of mathematical knowledge, however, translating this to their own educational practice in primary school might have provided them with more insight into the practical applicability and abilities of their students. This could be a worthwhile extension to the Mathematics in The News assignment when used again or in different educational contexts.

We suggest that Mathematics in The News is a useful context for school mathematics classes. By asking open questions teachers can use the rich context of news articles to prompt their students' mathematical thinking.

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